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AN EFFICIENT MODEL FOR CALCULATING VIBRATION FROM A RAILWAY TUNNEL BURIED IN A HALF-SPACE

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Abstract

Vibration from railway tunnels has a significant environmental impact on inhabitants of buildings near underground railway lines. The problem is of increasing importance because of the introduction of new underground lines in urban areas, increasing public sensitivity to noise and vibration, and the need to conform to increasingly stringent legislation.

There is a great need for railway-design engineers to model vibration from railway tunnels. A model is required to predict absolute levels of vibration from running trains and to assess the performance of vibration countermeasures before these are implemented. The model should be accurate and should not require substantial computational resources.

This paper presents a new model for calculating vibration from railway tunnels, which combines both accuracy and computational efficiency. The model is based on the assumption that the near-field displacement of the tunnel is not influenced by the existence of a free surface. Displacements at the tunnel-soil interface are calculated using a model of a tunnel wall in a full-space, known as the Pipe-in-Pipe (PiP) model. Green's functions for a two-and-a-half-dimensional elasto-dynamic full-space are then used to calculate the internal source in a full-space that would produce the same displacements at the tunnel-soil interface as calculated by the PiP model. The internal source is then used to calculate the far-field displacements based on Green's functions for a two-and-a-half-dimensional half-space.

The results and computation time of this model are compared with those of an alternative coupled Finite-Element-Boundary-Element (FE-BE) model that accounts for a tunnel wall in a half-space.

Eds.: J. Eberhardsteiner, H.A. Mang, H. Waubke

INTRODUCTION

Significant vibration in buildings near underground tunnels is attributed to moving trains. Vibration is generated at the wheel-rail interface due to irregularities of the tracks and wheels. The characteristics of the transmission path, i.e. the tunnel and its surrounding soil, determine the amount of vibration that reaches a building. Vibration can travel long distances, e.g. more than 200m in ground with soft clay and silt [6], causing annoyance to people and malfunctioning of sensitive equipment.

Vibration from underground railways can be isolated by reducing the railpad stiffness, using floating-slab tracks and/or supporting buildings on springs. There are other well known techniques which are described in the literature, see ref. [4] for example. Design engineers need an accurate and computationally efficient model to predict vibration from railway tunnels and assess the performance of these countermeasures. The model should be accurate due to the high financial cost of vibration countermeasures and the difficulty of retrospective replacement. It should also be computationally efficient as time is always a crucial factor when taking engineering decisions.

There are two popular approaches for calculating vibration from railway tunnels. The first is the Pipe-in-Pipe (PiP) model [3]. The tunnel wall and its surrounding infinite soil are modelled as two concentric pipes. The inner pipe represents the tunnel wall and is modelled using thin shell theory. The outer pipe, with its outer radius being set to infinity, represents an infinite soil with a cylindrical cavity and is modelled using elastic continuum theory. The PiP model is computationally efficient on account of the uniformity along and around the tunnel. The disadvantage of this model is that it does not account for a free surface or layering of the ground, and therefore it is more applicable to deep-bored tunnels.

The second approach is the coupled Finite-Element-Boundary-Element (FE-BE) model [1,2]. The tunnel wall is modelled using the Finite-Element method while the surrounding soil is modelled using the Boundary-Element method. This model is accurate, provided careful consideration is given to mesh discretization. The advantage of this method is the possibility of modelling a free surface and/or a multi-layered ground. It can also account for tunnel walls with non-circular geometries. The running time of this model has been greatly reduced in the last few years by taking advantage of periodicity in the longitudinal direction. The Floquet transform is incorporated by dividing the infinite model into repeating-units, and only one of these units is discretized. Even with this recent development, the model takes a long time to run and requires significant computational resources. It is useful for research purposes but still computationally expensive as a design tool.

This paper presents a variant to the PiP model which accounts for the free surface, thereby increasing accuracy without sacrificing computational efficiency. The model is based on the assumption that the near-field displacement of the tunnel is not influenced by the existence of the free surface. Displacements at the tunnel-soil interface are first calculated using the PiP model. The internal source in a full-space that would produce the same displacements at the tunnel-soil interface as calculated by the PiP model is then computed using Green's functions for a two-and-a-half-dimensional full-space. The internal source and Green's functions for a two-and-a-half-dimensional half-space are finally used to calculate the far-field displacements.

The new model is described further in the following sections. An outline of the model is first presented, followed by a comparison of the results and computation time with those of the equivalent coupled FE-BE model.

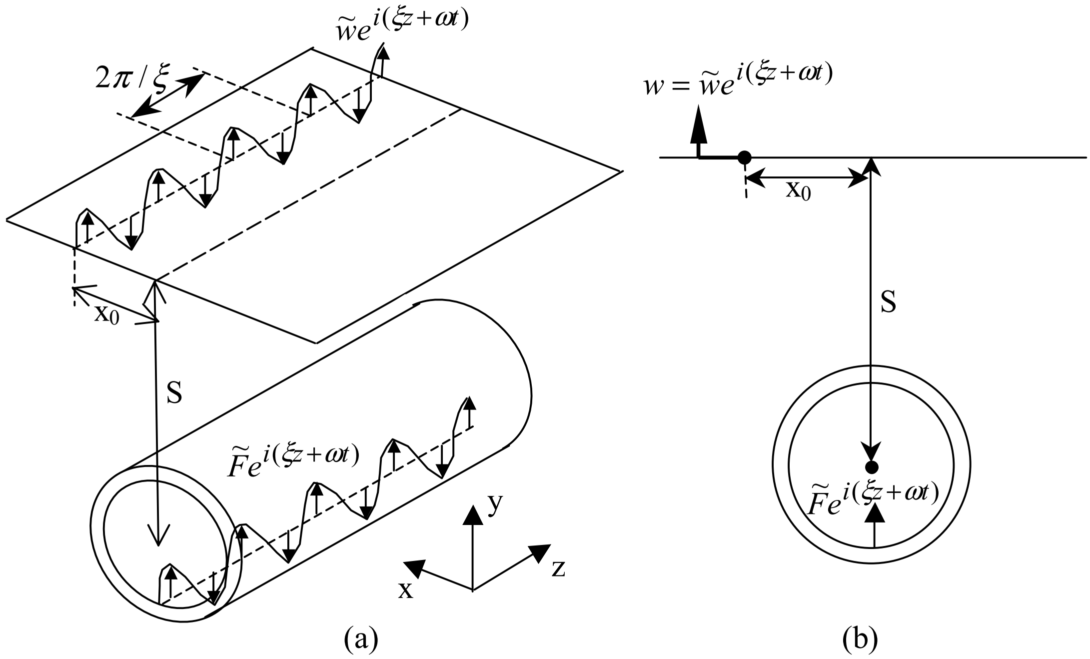

Figure 1: layout of the model showing the force-displacement lines in (a) three dimensions and (b) in the section at $z=0$.

OUTLINE OF THE MODEL

In this paper, the vertical displacement at the free surface due to a load applied on the tunnel invert is calculated. The load takes the form $F=\tilde{F}e^{i(\xi z+\omega t)}$ and the displacement takes the form $w=\tilde{w}e^{i(\xi z+\omega t)}$, as shown in Figure 1. These forms are the basis of the analysis in the wavenumber-frequency domain, which can be used to calculate the displacement for any sort of loading such as a concentrated harmonic load, see [5] for more details.

The displacement amplitude $\tilde{w}$ can be calculated as shown in the following subsections by: (1) calculating the displacements at the tunnel-soil interface using the PiP model; (2) using the Green's functions for a two-and-a-half-dimensional elasto-dynamic full-space to calculate the equivalent internal source in a full-space; and (3) using the Green's functions for a two-and-a-half-dimensional elasto-dynamic half-space to calculate the far-field displacement.

Calculating the displacements at the tunnel-soil interface

As mentioned before, it is assumed that the displacements at the tunnel-soil interface are not influenced by the existence of a free surface. The displacements $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_M$ due to the load $F = \tilde{F}e^{i(\xi z+\omega t)}$ are calculated using the PiP model, as shown in Figure 2. Each $\mathbf{u}_j$ (j=1, 2,…, M) is a vector that gives the longitudinal, horizontal and vertical displacements respectively and can be expressed as $\mathbf{u}_j = \tilde{\mathbf{u}}_j e^{i(\xi z+\omega t)}$. Note that M=5 is illustrated in the figure while a much larger value is required for better accuracy of the calculated displacement at the free surface, as will be seen later. As stated before, an infinite soil surrounding the tunnel is considered in the PiP model, which results in a tool that runs efficiently on personal computers (PCs) with no substantial memory requirements. A detailed description of the PiP model can be found in [3]. For improved accuracy, the tunnel wall is modelled in this work using elastic continuum theory, rather than the thin shell theory used in ref. [3].

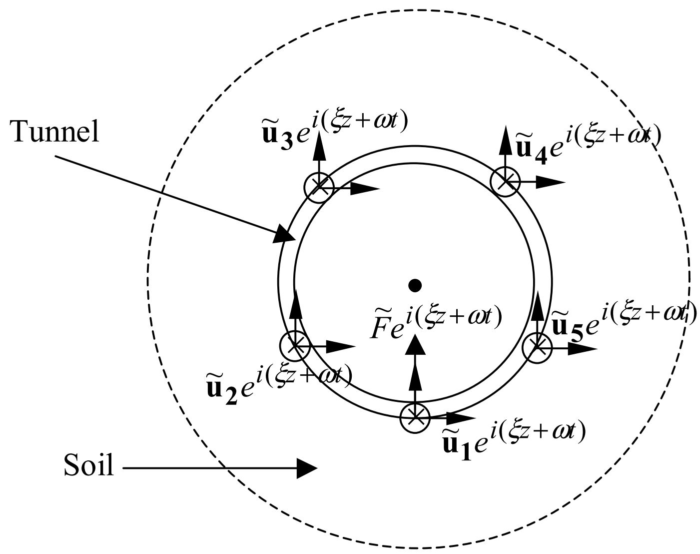

Figure 2: calculating the displacements at the tunnel-soil interface using the PiP model. ⊗ denotes the displacement component in the longitudinal direction, i.e. into the page.

Calculating the internal source

Consider a unit-magnitude line load applied in a full-space along the z direction of the form $e^{i(\xi z+\omega t)}$. The displacement at a line away and parallel to the line load is given by $\widetilde{G} \cdot e^{i(\xi z+\omega t)}$, where $\widetilde{G}$ is the Green's function for a two-and-a-half-dimensional elasto-dynamic full-space. Note that the value of $\widetilde{G}$ depends on: the properties of the full-space; the directions of the force and the measured displacement; the differences in the x and y coordinates between the force-displacement lines; and the wavenumber ξ and the excitation frequency ω. Analytical expressions for the full-space Green's functions are calculated by Tadeu and Kausel [7].

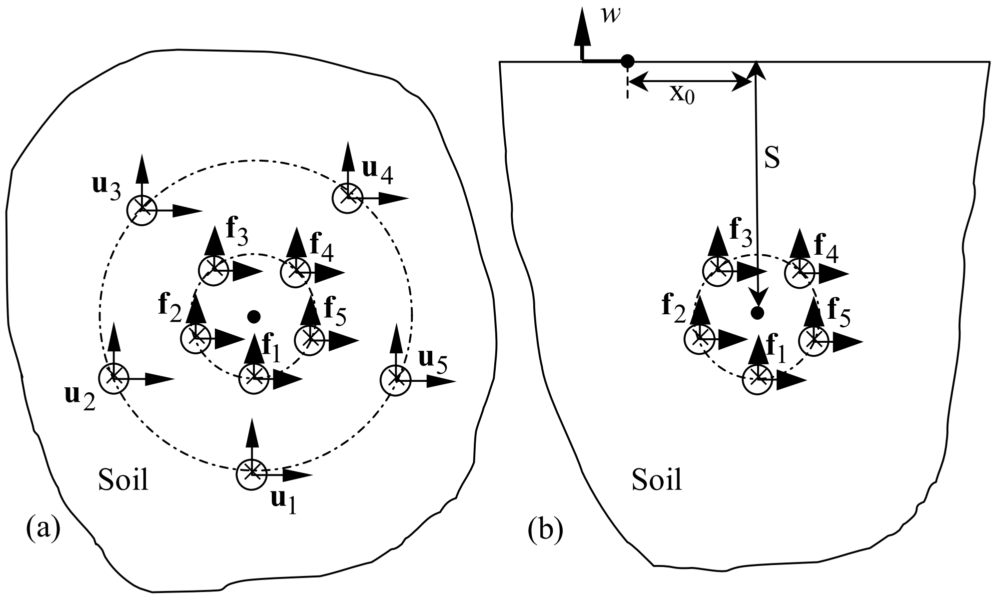

Figure 3: (a) calculating the equivalent internal source in a full-space, i.e. the values of $\tilde{\mathbf{f}}_1, \tilde{\mathbf{f}}_2, ..., \tilde{\mathbf{f}}_M$, and (b) using these forces with the Green's functions for a half-space to calculate the far-field displacements, i.e. at the free surface.

Having calculated the values of $\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \tilde{\mathbf{u}}_3, ..., \tilde{\mathbf{u}}_M$ from the PiP model, the equivalent internal source is calculated by using the Green's functions for a two-and-a-half-dimensional elasto-dynamic full-space. The internal source consists of M line loads positioned on a virtual cylinder in a full-space, as shown in Figure 3(a). The virtual cylinder has a radius less than the outer radius of the tunnel wall and each load has three force components, i.e. longitudinal, horizontal and vertical. Using Green's functions for a full-space, the forces $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, ..., \mathbf{f}_M$ ($\mathbf{f}_j = \tilde{\mathbf{f}}_j e^{i(\xi z+\omega t)}$) that produce the displacements $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, ..., \mathbf{u}_M$ ($\mathbf{u}_j = \tilde{\mathbf{u}}_j e^{i(\xi z+\omega t)}$) are calculated from the following relationship:

$$\begin{bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \\ : \\ : \\ \tilde{\mathbf{u}}_M \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{G}}_{11}^{full} & \tilde{\mathbf{G}}_{12}^{full} & : & : & \tilde{\mathbf{G}}_{1M}^{full} \\ \tilde{\mathbf{G}}_{21}^{full} & \tilde{\mathbf{G}}_{22}^{full} & : & : & \tilde{\mathbf{G}}_{2M}^{full} \\ : & : & : & : & : \\ : & : & : & : & : \\ \tilde{\mathbf{G}}_{M1}^{full} & \tilde{\mathbf{G}}_{M2}^{full} & : & : & \tilde{\mathbf{G}}_{MM}^{full} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{f}}_1 \\ \tilde{\mathbf{f}}_2 \\ : \\ : \\ \tilde{\mathbf{f}}_M \end{bmatrix} \tag{1}$$

where $\tilde{\mathbf{G}}_{jk}^{full}$ is a 3×3 matrix whose elements give the longitudinal, horizontal and vertical displacements at line j in the outer cylinder (with radius equal to the outer radius of the tunnel) due to the longitudinal, horizontal and vertical forces at line k in the inner cylinder (with radius less than the outer radius of the tunnel). It should be noted that the internal source calculations could be performed using Green's functions for a half-space. However, unlike the full-space Green's functions, analytical expressions are not available for the half-space functions, which must be calculated through numerical integrations. The method presented here takes advantage of the computational efficiency of the full-space functions by assuming that the free surface is sufficiently far from the tunnel so as not to have a significant influence on the near-field calculations.

Calculating the far-field displacement

Green's functions for a two-and-a-half-dimensional elasto-dynamic half-space are calculated by Tadeu et al. [8]. These are used to compute the far-field displacement from the following relationship:

$$\tilde{w} = [\tilde{\mathbf{G}}_{01}^{half} \quad \tilde{\mathbf{G}}_{02}^{half} \quad .. \quad .. \quad \tilde{\mathbf{G}}_{0M}^{half}][\tilde{\mathbf{f}}_1 \quad \tilde{\mathbf{f}}_2 \quad .. \quad .. \quad \tilde{\mathbf{f}}_M]^T \tag{2}$$

where $\tilde{\mathbf{G}}_{0k}^{half}$ is a 3×1 matrix whose elements give the vertical displacement at a line on the free surface due to the longitudinal, horizontal and vertical forces at line k of the cylindrical source, see Figure 3(b).

RESULTS

The displacements at the free surface due to a unit concentrated harmonic force applied at the tunnel invert at z=0 are presented here. The displacements are calculated first in the wavenumber domain using the technique described in the previous sections. Results are then transformed to the space domain, see [5] for a description of the method. The tunnel and the ground parameters used here are as follows: for the tunnel, an outer radius r_t = 3.0m, thickness h=0.25m, modulus of elasticity E_t= 50GPa, Poisson's ratio υ_t= 0.3, density ρ_t= 2500kg/m^3, and no damping. For the soil: a compression wave speed c_1= 944m/s, shear wave speed c_2= 309m/s, Poisson's ratio υ_s= 0.44, and a loss factor η=0.06 associated with both Lame's constants. The results in Figures 4 and 5 are calculated with a value M=40.

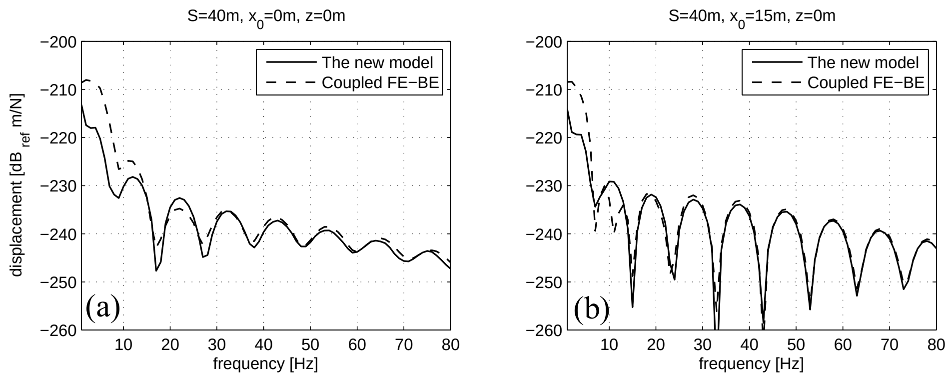

Figure 4: displacement at the free surface due to a concentrated load applied at the invert of a tunnel with depth 40m (a) displacement at x_0=0m, (b) displacement at x_0=15m. See Fig. 1 for definitions of S and x_0.

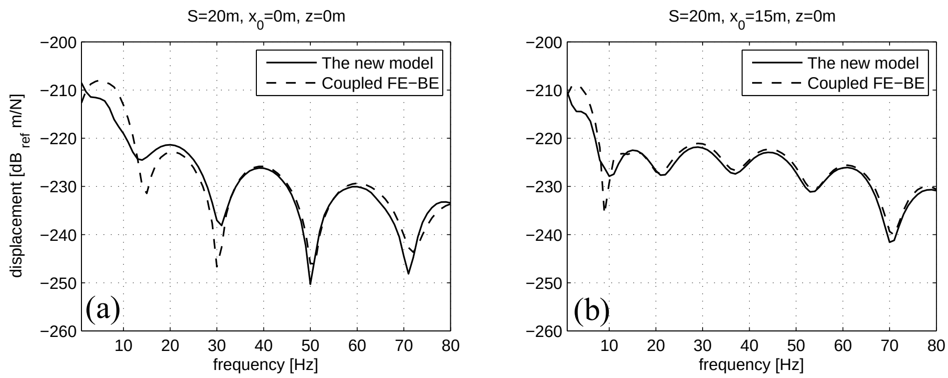

Figure 5: displacement at the free surface due to a concentrated load applied at the invert of a tunnel with depth 20m (a) displacement at x_0=0m, (b) displacement at x_0=15m.

It can be seen from Figures 4 and 5 that the results from the new model agree well with the ones from the coupled FE-BE model for frequencies above 15Hz and tunnel depths greater than or equal to 20m. The running time for the new model to produce the results in Figure 4 is approximately 3hrs on a PC with 1GB RAM and 2.4GHz processor. The same results are calculated using the coupled FE-BE method in approximately 17hrs on a HPC (high performance cluster at K.U. Leuven). This demonstrates clearly the computational efficiency of the new method. Work is in progress to determine the limitations of the new model, such as the minimum depth of the tunnel at which the existence of a free surface has a significant effect on the near-field displacements.

CONCLUSIONS

An efficient model for calculating vibration from a railway tunnel buried in a half-space has been presented in this paper. This model is validated against the coupled FE-BE method. The new model is computationally efficient. It can therefore be used as an engineering tool to predict vibration from underground railway trains and to assess the performance of vibration countermeasures.

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