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# AN INSERTION LOSS MODEL FOR EVALUATING THE PERFORMANCE OF FLOATING-SLAB TRACK FOR UNDERGROUND RAILWAY TUNNELS

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## Abstract

This paper discusses a comprehensive three-dimensional model of a deep underground railway tunnel. The model consists of Euler-Bernoulli beams to account for the rails and the slab track. These are coupled to an infinite continuum (pipe-in-pipe) model, which accounts for the tunnel wall and the cylindrical cavity within an infinite soil domain. The track slab is coupled to the tunnel wall using slab-bearings along several lines. The coupling is performed in the wave number domain and results for soil displacements and velocities are obtained by transformation to the space domain. Random vibration theory is used to predict vibration levels in the soil due to an empirical rail roughness input. The model shows the importance of soft slab-bearings in decreasing the vibration levels by confining the energy in the slab. It also shows the effect of distribution of slab-bearings on the vibration isolation performance.

## Introduction

One of the major sources of ground-borne vibration is the running of trains in underground railway tunnels. This kind of vibration is categorized as a low-frequency problem, where the frequency of interest ranges from 0-200Hz. The effect of ground-borne vibration is annoyance to people rather than damage to buildings, see the report by Office for Research & Experiments of the International Union of Railways [1].

Many methods are used to decrease vibration levels in structures. Some depend on isolating the vibration at the receiver, for instance, by using rubber bearings at the buildings' foundations. Others depend on disrupting the passage of vibration, for instance, by constructing deep trenches. One of the most effective methods is to isolate vibration at the source itself. A good example of source isolation is the use of floating-slab track. The idea is to support the main track slab on rubber blocks, steel springs or a continuous sheet of rubber.

A typical cross-section of the underground-tunnel is shown in Figure 1 and consists of the in-tunnel structure (rails, rail-pads, floating-slab). The floating-slab is coupled via slab-bearings to the tunnel wall, which lines a cavity in the soil.

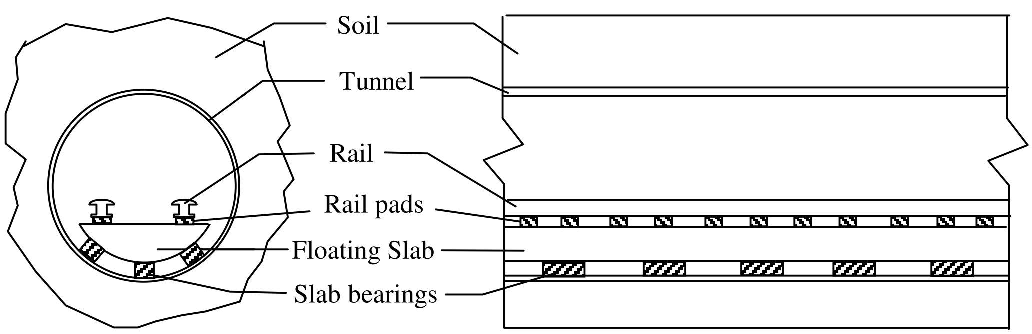


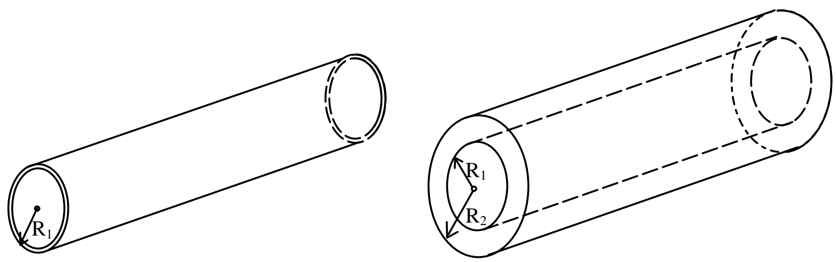

*Figure 1: Layout of an underground-tunnel showing the different structural components (in this figure: the floating-slab is mounted on three lines of discrete slab-bearings)*

Existing models to predict the vibration from the running trains range from simple models, based on one degree of freedom or Winkler beam theory, to more complicated models based on numerical techniques such as the finite element method. The simple models lack accuracy and do not account for the dispersion characteristics of the tunnel wall and the ground, while the numerical models require long computation times.

This paper is extending a three-dimensional model of a deep underground railway tunnel as developed by Forrest [2]. Forrest only considered radial loading on the tunnel invert; (that is the lower section of the tunnel which support the track), where the present study includes tangential loading. Convergence problems encountered by Forrest when using very soft slab-bearings are addressed by introducing damping to the rail-slab system to satisfy the requirements of the discrete Fourier transform, for more details see Hussein [3]. Two cases of slab supports are considered: uniform continuous support (slab poured on a mat of rubber or directly onto the tunnel invert) and discrete support (slab mounted on discrete springs or rubber blocks). Predictions of soil root mean square (rms) velocity are calculated, which are more appropriate for assessing in the human perception than rms displacement results.

## Outline of the model

A three-dimensional analytical model for the underground tunnel is obtained by:
1) modelling the tunnel wall(Figure 2.a) and the surrounding soil (Figure 2.b) (Pipe-in-pipe model);
2) modelling the track, that is, the rails and the floating-slab;
3) coupling the two models directly or via slab-bearings.




(a) (b)

*Figure 2: Pipe-in-pipe model: (a) tunnel wall of outer radius $R_1$ modelled as a thin cylindrical shell. (b) Surrounding soil modelled as a thick cylindrical shell of internal radius $R_1$ and external radius $R_2$ set to infinity.*

The first two parts are each formulated to provide input-output equations. These equations are established in the wave number domain; this is equivalent to a system of output displacements resulting from input forces with longitudinal and angular spatially harmonic distributions.

The tunnel wall is modelled as a thin cylindrical shell. The surrounding soil is modelled as a thick cylindrical shell with internal radius equal to the outer radius of the tunnel wall and an infinite external radius. This assumes an infinite homogenous domain around the tunnel, which is a good model for deep tunnels. These two cylinders are coupled by considering the boundary condition for displacements and stresses at the interface between them.

The rails and slab are modelled using Euler-Bernoulli beam theory, in which shear deformation and rotary inertia are neglected. This is acceptable in modelling of ground- borne vibration where small frequencies (long wavelengths compared to the beam cross-sectional dimensions) are considered.

The track is coupled to the tunnel invert either directly or via slab-bearings. The slab-bearings can be included between the slab and the tunnel invert as discrete supports or continuous sheet. The coupling is performed in the wave number domain by satisfying equilibrium and compatibility for slab-bearings.

In the following discussion, four kinds of slab supports are considered as shown in Figure 3. The slab-bearings in these models have normal as well as shear stiffness. The shear stiffness is taken as half of the normal stiffness of the bearings; the effect of changing this ratio can also be investigated through these models. For each model in Figure (3), the normal and tangential stiffness are computed for a certain vertical natural frequency of the slab. This is computed by using the two-dimensional models analogous to these in Figure (3) with a rigid tunnel wall. For instance, the normal

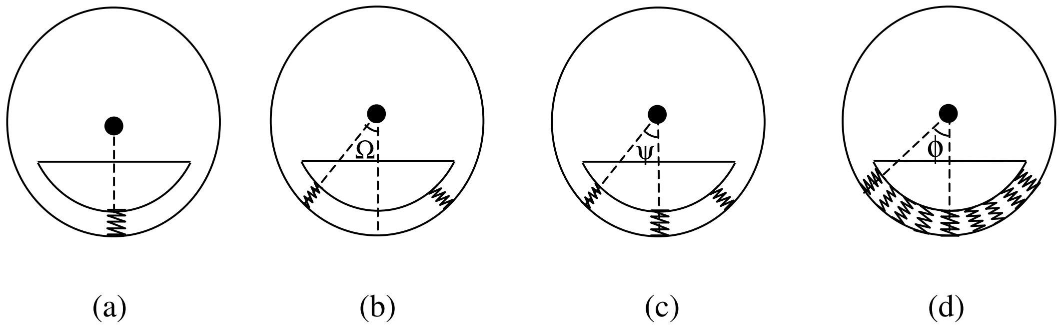



*Figure 3: Coupling of the slab to the pipe-in-pipe model via: (a) single line, (b) two lines, (c) three lines, and (d) uniform continuous support. Rails are not shown for clarity.*

stiffness k in the first model is computed for a prescribed natural frequency $f_n$ from the relationship: $f_n=(1/(2\pi))\sqrt{(k/m)}$, where m is the mass of the slab per meter length.

## Effectiveness of Slab Floating using Random Vibration Theory

In this section the model in Figure 4(b) is used to predict some statistical information for the soil response when an infinite train passes through the tunnel. The purpose is to compare the performance of the floating-slab using different slab-bearings with the performance of the slab directly supported to the tunnel.

The model in Figure 4(b) was described by Forrest [2] for a single line of slab-bearings and modified in this work for the multi-lines case. The input to this model is an empirical rail-wheel roughness exciting at infinite set of axles with spacing L. The output is measured at a point A in Figure 4(b), which can be taken at any position around the tunnel. A white noise input is considered to investigate the general behavior of the system through the output spectral density at any frequency. This output is then weighted with empirical roughness input calculated by Frederich [4] for a real rail roughness input. This model considers two correlated sides (right wheels and rail with left wheels and rail), and thus accounts only for the bending case of the slab.

## Results and Discussion

Figure 5 shows the output power spectral density at a point 20m above the tunnel centre for a white noise input. Results above the tunnel and on its side are important where some buildings' foundations are expected to exist. This figure is of particular importance as it enables investigating the system for general roughness input (see (3) in Figure 4(a)). In Figure 5 it can be seen that the isolation performance starts at frequency $\sqrt{2}f_n$, as for the case of a single-degree-of-freedom model. This is particularly true for slabs with soft slab-bearings (isolation starts when each curve intersects the direct fixation curve and then decreases). Direct fixation results are calculated by setting the vertical natural frequency of the slab to infinity. Resonance

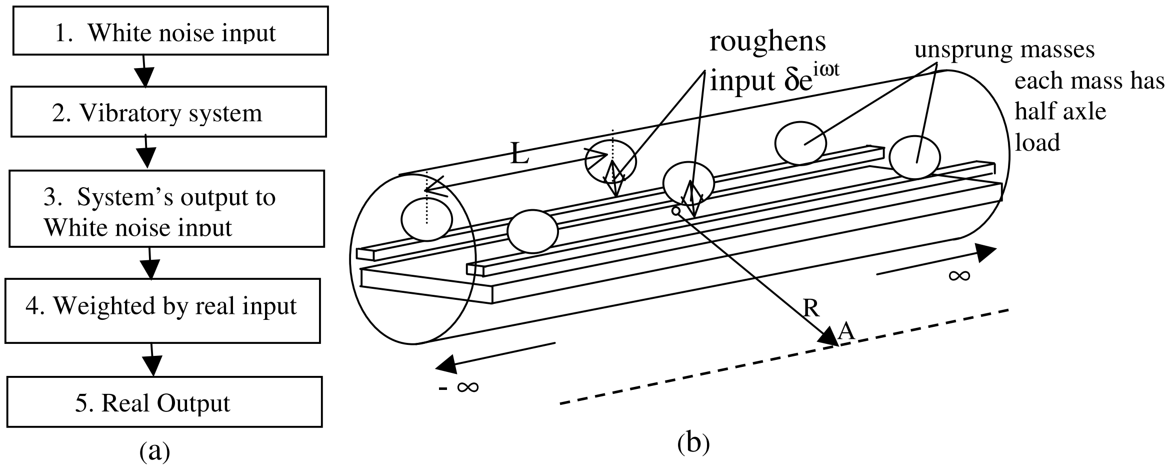



*Figure 4: Schematic diagram showing: (a) the main parts in performing a random vibration analysis and (b) the vibratory system of an underground-tunnel (only part of the longitudinal infinite model is shown). The input is shown for the middle axle in (b) (the two masses in the middle) with the same magnitude ($\delta$) to account only for slab bending.*

of these curves corresponds to excited cross-sectional wave-numbers modes around the tunnel at these frequencies.
Frederich's empirical power spectral density for the rail roughness is give by:

$$S_{\delta}(f) = \frac{a}{v(b + f/v)^3} \qquad (1)$$

where: a and b are constants computed by Frederich for different conditions of the rail. $f$ is the frequency and $\nu$ is the train speed. The important remark about this function is that it gives larger weights for lower frequencies.
Figure 6(a) shows the rms displacement levels along a horizontal line (perpendicular to the tunnel direction) 20m above the tunnel centre for Frederich's roughness input when the slab is directly supported to the tunnel (bearings are distributed around the slab circumference with infinite normal and shear stiffness). Figure 6(b), 6(c) and 6(d) show the insertion gains for the case of 3 lines, 2 lines and 1 line of slab-bearings.
The insertion gain is computed by the following relationship:

$$\text{Insertion gain}\,[dB] = 20\log_{10}\left(\frac{Z_{f_n Hz}}{Z_{Dirext}}\right) \qquad (2)$$

Where $Z_{fnHz}$ is the Soil rms response (displacement or velocity) when using slab with $f_n$ isolation frequency. $Z_{Direct}$ is the soil rms response when the slab is poured directly onto the tunnel (continuous support with infinite stiffness).

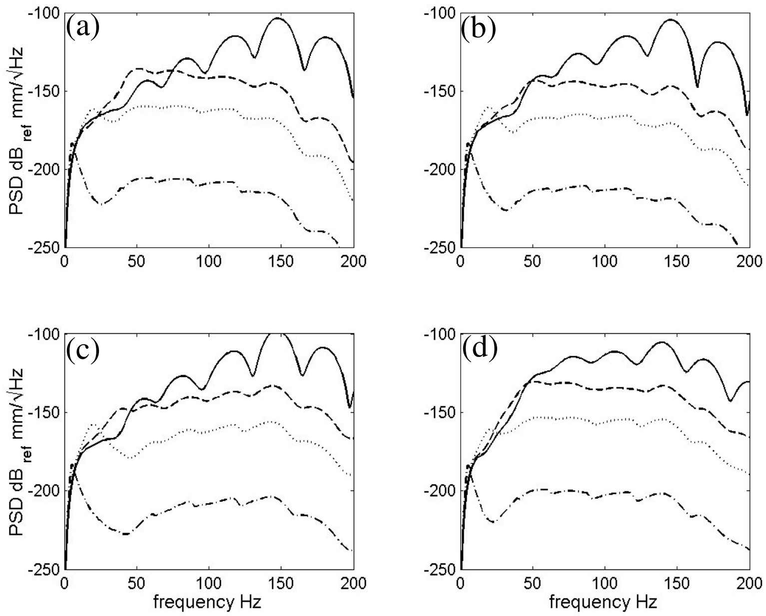



*Figure 5: Power spectral density output at a point 20 m directly above the tunnel centre for white noise roughness input at the rail for a slab attached to the tunnel via: (a) continuous layer, (b) three lines, (c) two lines and (d) one line. Slab bearings corresponding to direct fixation (solid), and isolation frequency of 40Hz (dashed), 20Hz (dotted) and 5Hz (chained) are considered.*

*For instance, the solid curve in (c) is for a slab supported directly (infinite stiffness) to the tunnel via two lines as in figure 3(b). The following parameters are considered for the analysis: for the rails: flexural rigidity $EI=5x10^6Pa.m4$, mass per unit meter100kg/m, loss factor $\eta_{EI}=0.02$, for the rail-pads: stiffness $k=200x10^6N/m^2$, loss factor $\eta_k=0.3$, for the slab: flexural rigidity $EI=1430x10^6Pa.m^4$, mass 3500kg/m, loss factor $\eta_{EI}=0.05$, axle mass $m_a=500kg$, spacing between axles L= 20m, for the tunnel: elasticity modulus $E=50x10^9Pa$, Poisson's ratio $\nu=0.3$, density $\rho=2500kg/m^3$ tunnel mean radius a=3.0 m, tunnel thickness h=0.25m, for the soil, elasticity modulus $E=550x10^6Pa$, Poisson's ratio $\nu=0.44$, density $\rho=2000kg/m^{3,}$ Lame's constants $\lambda=1.4x10^9Pa$ $G=191x10^6Pa$. $\phi=30°$, $\psi=30°$, $\Omega=30°$, (see Figure 3).*

To compute the total rms displacement levels, results at (b), (c), and (d) should be added to the results in (a). Referring to Figure 5, for 5Hz slab-bearings, PSD results are very small above 20Hz (below –200dB), which do not contribute in the summation to compute the rms values. Below 20Hz, PSD values are the same irrespective of number of slab-bearing lines (lower mode of the tunnel-soil model is dominant). This explains why the rms results for soft-slab bearing are nearly similar (see Figure 6 (b), (c), and (d)).

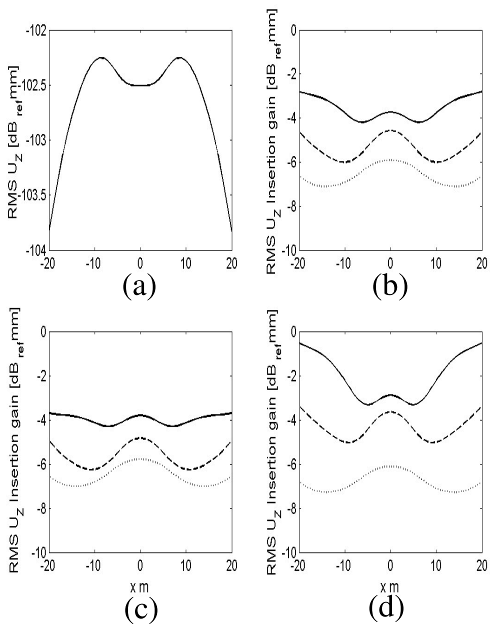



*Figure 6: Vertical RMS displacement for a horizontal line (perpendicular to the tunnel direction) 20 m above the tunnel centre using roughness input of a rail in an average condition. (a) RMS displacement for continuous slab-bearings layer of infinite stiffness (Direct fixation). (b), (c), and (d) are displacement insertion gain at the same line for slab attachment through three lines, two lines and one line respectively. Slab bearings corresponding to isolation frequency of 40Hz (solid), 20Hz (dashed) and 5Hz (dotted) are considered. Parameters used as given in figure (5). Frederich's roughness function is used for 40km/hr train speed and a rail in an average condition; in this case: a and b are* $1.31x10^{-2}$ $mm^2.(1/m)^2$ *and* $b=2.94x10^{-2}$

Much improvement is obtained for the velocity rms by floating the slab (more than 30dB when using slab-bearings with 5 Hz isolation frequency), see Figure (7). This is because the PSD results are now weighted by $f^2$, see Newland [5], which makes the higher frequencies in the direct fixation case greatly contribute in the summation for the rms , while PSD results at high frequencies for the soft slab-bearings are very small even after being weighted with $f^2$. This increases the difference between rms

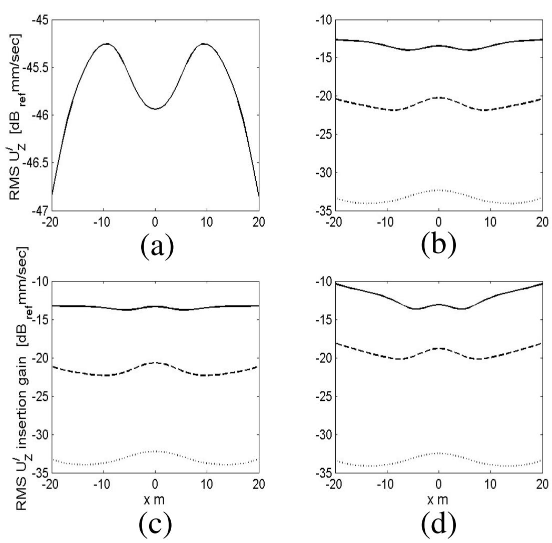



*Figure 7: Vertical RMS velocity for a horizontal line 20 m above the tunnel centre. (a) RMS velocity for continuous slab-bearings layer of infinite stiffness (Direct fixation). (b), (c), and (d) are velocity insertion gain at the same line for slab attachment through three lines, two lines and one line respectively. Slab bearings corresponding to isolation frequency of 40Hz (solid), 20Hz (dashed) and 5Hz (dotted) are considered. Parameters are given in Figure (5).*

results for the slab directly supported to the tunnel and the one supported via soft slab-bearings. Even much better isolation is expected for the acceleration rms results due to the weight $f^4$ with the displacement PSD, see Newland [5].

## Conclusions and Future Work

The results presented in this paper, based on an empirical input function at the rail, show better vibration isolation when using softer slab-bearings. For very soft slab-bearings, the effect of distributing the supports has no influence on the vibration levels around the tunnel. For stiffer slab-bearings the tunnel-soil-coupling interaction is strong and vibration results are more dependent on dispersion characteristic of the tunnel-soil system.

Insertion gain rms results are given for certain points in the soil. This does not completely reflect the general behaviour of the system, as vibration levels can considerably change from point to point in the soil. Also it does not give the proper input needed once a building's foundation is connected to any of these points. A more appropriate method is to use the power flow insertion gain, as developed by Talbot and Hunt [6] for assessing building isolation. Also, in order to have a more realistic model, the torsion effect of the slab should be included, which is now under progress.

## Acknowledgement

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